Analysis of Covariance
Linear Regression

Model:

$$Y_i = \mu + \beta(X_i - \bar{X}) + \varepsilon_i$$

Where: μ = overall mean

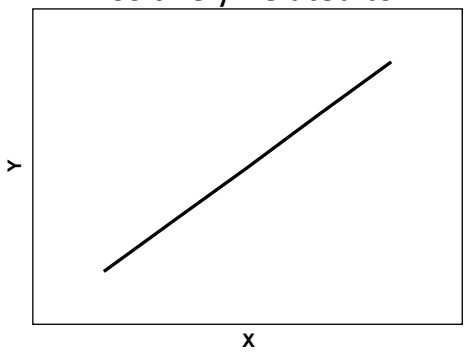
β = regression coefficient

X_i = is the independent variable

ε_i = error associated with measurement Y_i

Analysis of Covariance
Linear Regression

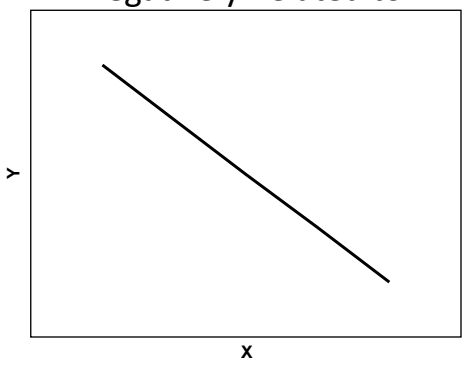
Y Positively Related to X



Analysis of Covariance

Linear Regression

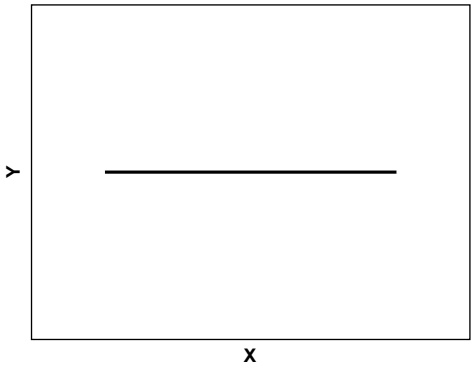
Y Negatively Related to X



Analysis of Covariance

Linear Regression

Y Not Related to X



Analysis of Covariance

Covariance

$$\sigma_{XY}^2 = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{n-1}$$

Analysis of Covariance

Correlation

$$\rho_{12} = \frac{\sigma_{12}}{\sigma_1 \sigma_2}$$

$$r = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y}) / (n-1)}{\sqrt{\sum (X_i - \bar{X})^2 / (n-1)} \sqrt{\sum (Y_i - \bar{Y})^2 / (n-1)}}$$

Analysis of Covariance
Model for CRD

$$Y_{ij} = \mu + T_i + \beta(X_{ij} - \bar{X}..) + \varepsilon_{(i)j}$$

Where: μ = overall mean
T_i = effect of i^{th} treatment
β = regression coefficient of Y on X
X_{ij} = is the covariate or concomitant variable
$\varepsilon_{(i)j}$ = effect of ij^{th} plot

Analysis of Covariance
Model for RCBD

$$Y_{ij} = \mu + B_i + T_j + \beta(X_{ij} - \bar{X}..) + \varepsilon_{ij}$$

Where: μ = overall mean
B_i = effect of i^{th} block
T_j = effect of j^{th} treatment
β = regression coefficient of Y on X
X_{ij} = is the covariate or concomitant variable
ε_{ij} = effect of ij^{th} plot (really the interaction term BT_{ij})

Analysis of Covariance

Uses

- Control experimental error and improve precision
- Adjust treatment means
- Assist in interpretation of the data
- Estimate values of missing observations

Analysis of Covariance

Blocking vs. Covariate

	ANOVA	ANCOVA
Conditions	Variation structured	Variation random
Advantages	• Requires no additional measurement • Done during design and layout	• Requires no knowledge of spatial relationship • Can be used after the fact

Analysis of Covariance
Kenaf Variety Trial

Block \ Plot	1	2	3	4	5	6	7	8
100	Dowling	Evergl71	Gregg	SF459	Evergl41	Whitten	T2	PF2
200	Gregg	SF459	Dowling	Evergl41	T2	Whitten	Evergl71	PF2
300	T2	PF2	Dowling	Evergl71	Gregg	Evergl41	Whitten	SF459
400	Evergl71	Dowling	Whitten	PF2	T2	Gregg	SF459	Evergl41

Analysis of Covariance
Kenaf Variety Trial

X = Plant Population

Block \ Plot	1	2	3	4	5	6	7	8
100	81.8	64.6	64.6	43.1	56.0	47.4	103.3	68.9
200	60.3	68.9	34.4	51.7	51.7	25.8	34.4	64.6
300	64.6	111.9	86.1	81.8	68.9	68.9	30.1	25.8
400	56.0	103.3	86.1	77.5	81.8	81.8	38.8	68.9

$\bar{X} = 64.178$ (k/ha)

Analysis of Covariance

Kenaf Variety Trial

ANOVA

Source	DF	Type I SS	Mean Square	F Value	Pr > F
Block	3	25.89921503	8.63307168	3.8	0.0254
Variety	7	72.90662239	10.41523177	4.59	0.003
Error	21	47.6776157	2.2703627		
Total	31	146.4834532			

ANCOVA

Source	DF	Type I SS	Mean Square	F Value	Pr > F
Block	3	25.89921503	8.63307168	4.90	0.0103
Variety	7	72.90662239	10.41523177	5.91	0.0008
Population	1	12.41884147	12.41884147	7.04	0.0152
Error	20	35.2587743	1.7629387		
Total	31	146.4834532			

Analysis of Covariance

Adjustment of Treatment Means

$$Y_i' = \overline{Y}_i - \beta(\overline{X}_i - \overline{X}..)$$

Where:

$\overline{Y}_i$ = mean Y of i^{th} treatment

$\overline{X}_i$ = mean X of i^{th} treatment

$\overline{X}..$ = overall X mean

Analysis of Covariance

Adjustment of Treatment Means

$$Y_i' = \bar{Y}_i - \beta(\bar{X}_i - \bar{X}..)$$

$$7.8 = 8.3 - 0.04(76.4 - 64.18)$$

Where:

$\bar{Y}_i$ = mean yield of 'Dowling'

$\bar{X}_i$ = mean population of 'Dowling'

$\bar{X}..$ = overall population mean

Analysis of Covariance

Least Square Means

Variety	Mean	Population	Adjusted Mean
Dowling	8.3	76.4	7.8
Evergl41	8.0	61.4	8.1
Evergl71	7.8	59.2	8.0
Gregg	8.3	68.9	8.2
PF2	5.6	80.7	4.9
SF459	7.2	44.1	8.0
T2	11.4	75.3	10.9
Whitten	7.7	47.4	8.4

Population Mean = 64.18 (k/ha) β = 0.040

Analysis of Covariance

SE for Adjusted Treatment Means

$$S_{\bar{d}} = \sqrt{MSE\left(\frac{1}{r_1}+\frac{1}{r_2}+\frac{(\bar{X}_1-\bar{X}_2)^2}{\sum_{i=1}^{t}\sum_{j=1}^{r}(X_{ij}-\bar{X}_i.)^2}\right)}$$

Analysis of Covariance

SE for Adjusted Treatment Means

$$S_{\bar{d}} = \sqrt{1.7629387\left(\frac{1}{4}+\frac{1}{4}+\frac{(76.4226-61.3530)^2}{12267.05}\right)} = 0.956$$

Analysis of Covariance

SE for Adjusted Treatment Means

RCBD

The GLM Procedure
Least Squares Means

var	yield LSMEAN	Standard Error
Dowling	8.3081320	0.7533861
Evergl41	8.0027920	0.7533861
Evergl71	7.7768600	0.7533861
Gregg	8.3446720	0.7533861
PF2	5.5787760	0.7533861
SF459	7.2125760	0.7533861
T2	11.3765400	0.7533861
Whitten	7.6985160	0.7533861

ANCOVA

The GLIMMIX Procedure
Least Squares Means

Kenaf Variety	Estimate	Standard Error
Dowling	7.8139	0.6895
Evergl41	8.1168	0.6653
Evergl71	7.9778	0.6682
Gregg	8.1546	0.6677
PF2	4.9107	0.7100
SF459	8.0218	0.7305
T2	10.9258	0.6853
Whitten	8.3774	0.7115

Analysis of Covariance

Assumptions

The covariate X is:

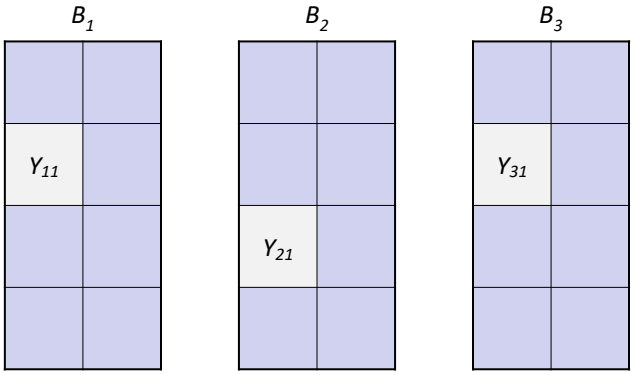
- Fixed
- Measured without error
- Independent of treatment effects

The relationship between Y and X is linear

The relationship between Y and X is the same for all treatments

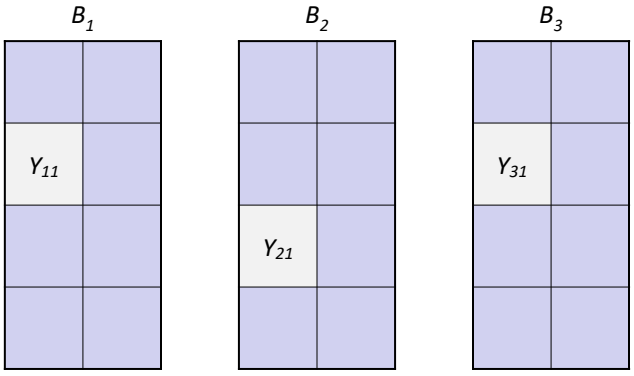
Analysis of Covariance

Nearest Neighbor Analysis (Papadakis Method)



Analysis of Covariance

Nearest Neighbor Analysis



For Treatment 1, in Plot 2 of Block 1:

$$d_{ij} = Y_{ij} - \overline{Y}_{.i} = Y_{11} - \overline{Y}_{.1} \qquad \overline{Y}_{.1} = \frac{Y_{11} + Y_{21} + Y_{31}}{3}$$

Analysis of Covariance

Nearest Neighbor Analysis

B_1

d_1	d_5
X	d_6
d_3	d_7
d_4	d_8

For Plot 2 in Rep 1:

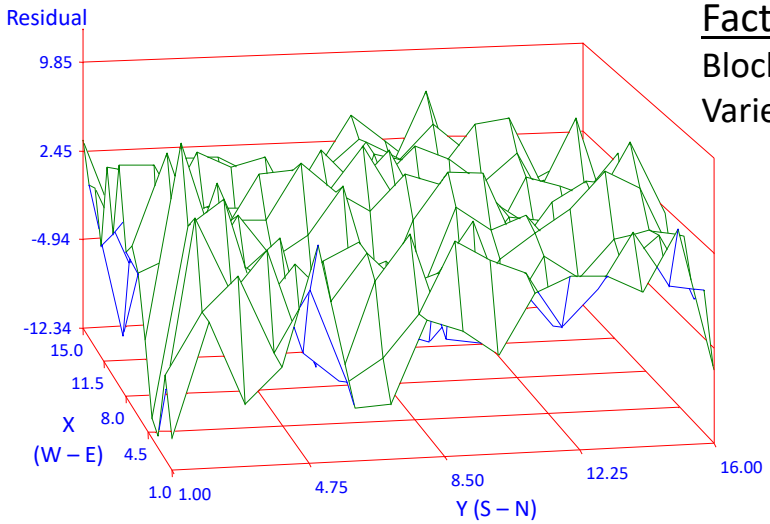
$$X_1 = \frac{d_1 + d_3 + d_6}{3}$$

or

$$X_2 = \frac{d_1 + d_3 + d_5 + d_6 + d_7}{5}$$

Nearest Neighbor Analysis

Wheat Variety Trial Example



Factors:

Blocks 4

Variety 60

Nearest Neighbor Analysis

Wheat Variety Trial Example

ANOVA

Source	DF	Type III SS	Mean Square	F Value	Pr > F
BLOC	3	143.973613	47.991204	2.79	0.0421
ENTRY	59	1248.015573	21.152806	1.23	0.1538
Error	177	3045.886187	17.208397		

ANCOVA

Source	DF	Type III SS	Mean Square	F Value	Pr > F
ENTRY	59	1219.628330	20.671667	1.44	0.0361
X1	1	539.348348	539.348348	37.58	<.0001
X2	1	46.223292	46.223292	3.22	0.0744
Error	178	2554.917794	14.353471		